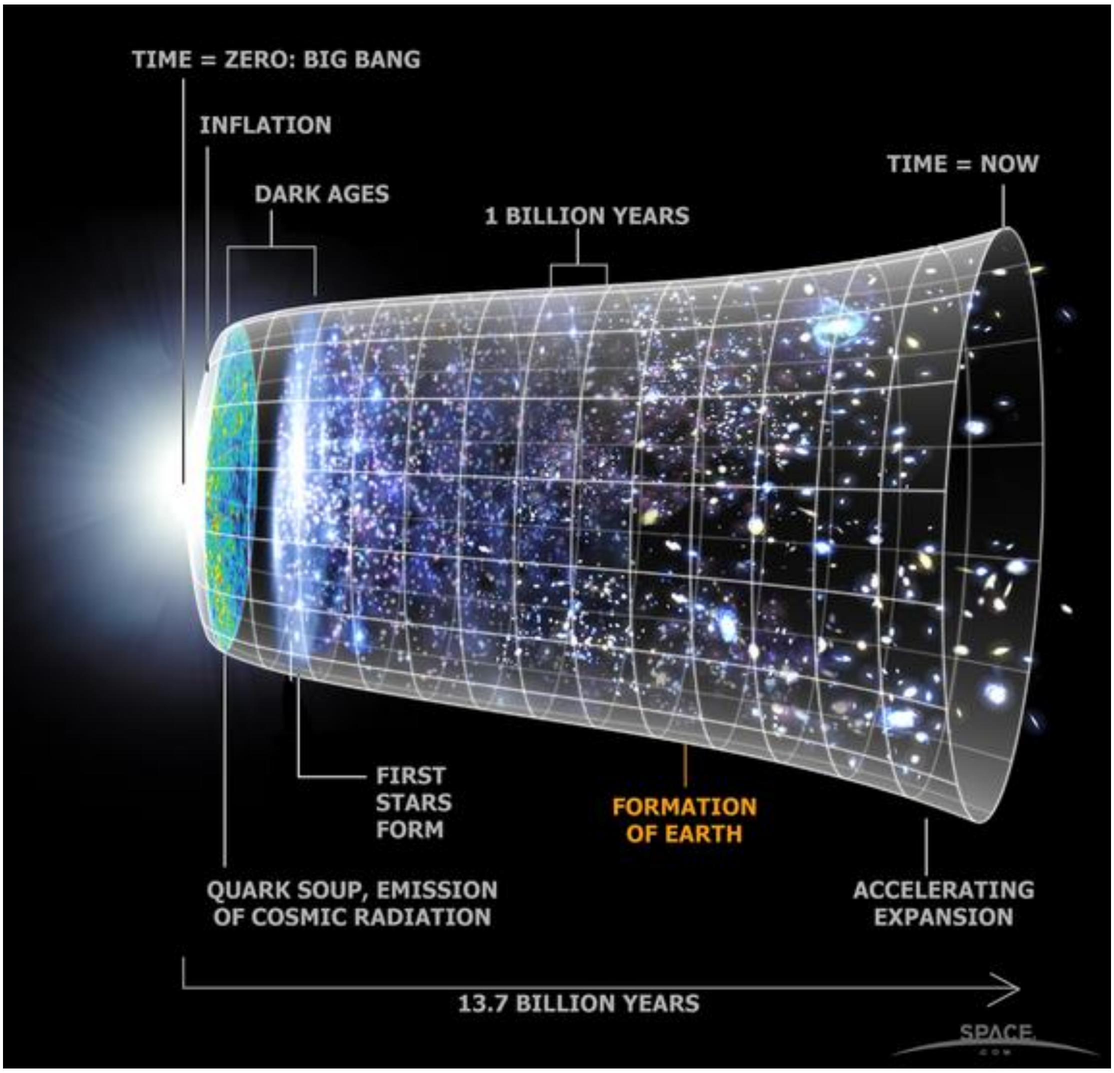
The universal theory

K.P.ANASTASIADIS- PHD COMPUTERIZED ASTROPHYSICS

Relativity and quarks



FORMULATION OF PRINCIPLES

MINKOWSKI

SPACE

LORENTZ

FACTOR

POINCARE

KARL SWARZCHILD

METRIC

RADIUS

PAUL DIRAC

FERMIONS

HAMILTON

POISON

SCHRODINGER

PODOLSKY

ROSEN

BRIDGE

ALBERT EINSTEIN

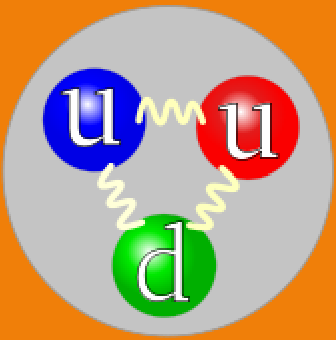
GENERAL RELATIVITY THEORY

BOSE

STATISTICS

CONDENSATE

QUARK



MAX PLANCK

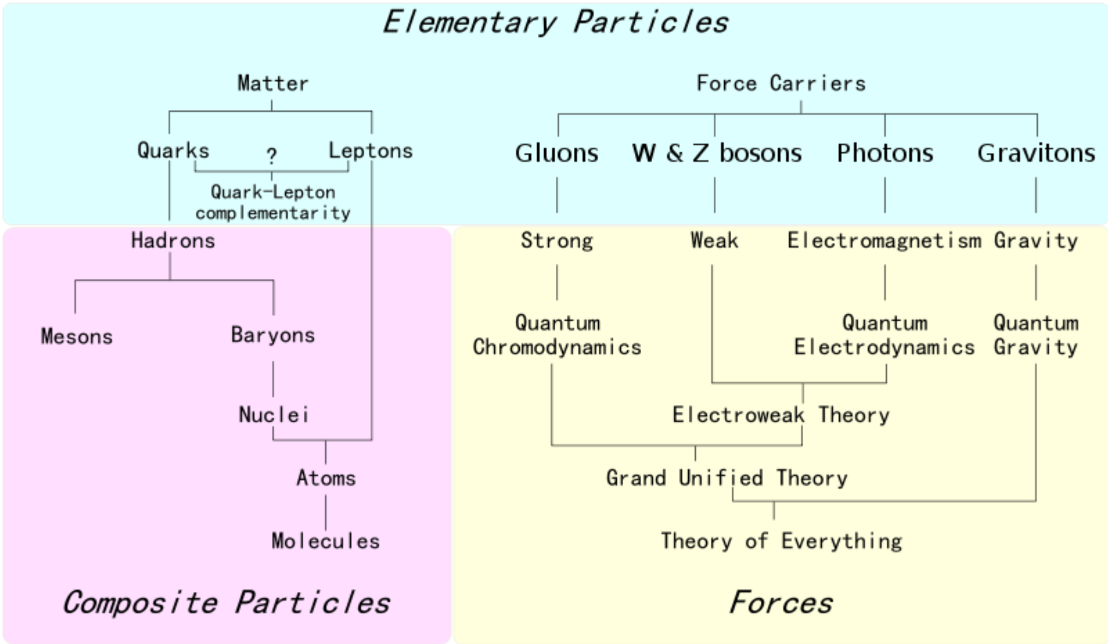
Constant

$$h = 6.626\ 069\ 57(29) \times 10^{-34}\ \mathrm{J \cdot s} = 4.135\ 667\ 516(91) \times 10^{-15}\ \mathrm{eV \cdot s}.$$

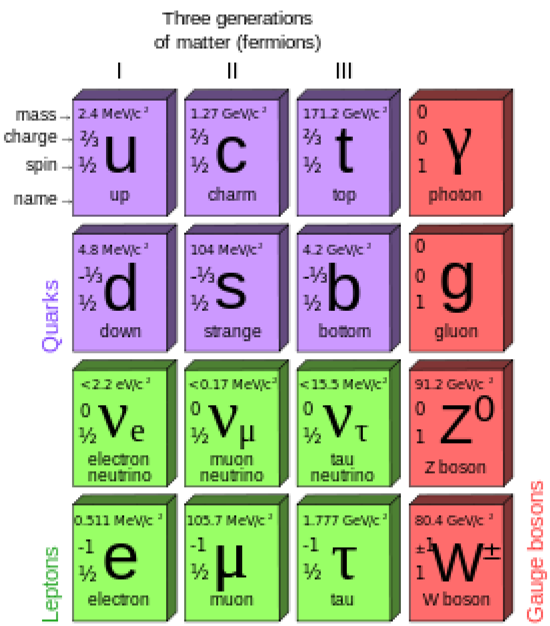
λv = c

Λ=WAVELENGTH, V=FREQUENCY, C=SPEED OF LIGHT

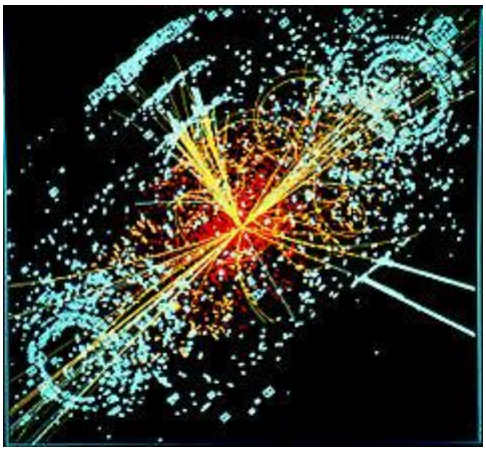
ELEMENTARY PARTICLES



1. Fermions
2. Leptons
3. Muons
4. Bosons
5. Hadrons
6. Mesons
7. Baryons
8. Gluons
9. Gravitons



HIGGS BOSON



SPIN

Angular momentum

CLASSIFICATION OF PARTICLES

All particles other than field particles can be classified into two broad categories,

hadrons and leptons, according to their interactions.

HADRONS

Particles that interact through the strong force are called hadrons. The two classes of hadrons, mesons and baryons, are distinguished by their masses and spins.

MESONS all have spin 0 or 1, with masses between that of the electron and that of the proton. All mesons are known to decay finally into electrons, positrons, neutrinos, and photons. The pion is the lightest of known mesons; it has a mass of approximately 140 MeV/c^2 and a spin of 0. Another is the K meson, with a mass of approximately 500 MeV/c^2 and a spin of 0.

BARYONS, the second class of hadrons, have masses equal to or greater than the proton mass (*baryon* means “heavy” in Greek), and their spins are always odd half-integer values (, , , etc.). Protons and neutrons are baryons, as are many other particles. With the exception of the proton, all baryons decay in such a way that the end products include a proton. For example, the baryon called the __ hyperon first decays to the _0 baryon and a __ in about 10_10 s. The _0 then decays to a proton and a __ in approximately 3 _ 10_10 s.

It is important to note that hadrons are composite particles, not point particles, and have a measurable size of about 1 fm (10_15 m). Hadrons are composed of more elemental units called *quarks*, which are believed to be truly structureless point particles. Mesons consist of two quarks and baryons of three. For now, however, we defer discussion of the ultimate constituents of hadrons to Section 15.9 and continue with our empirical classification of particles.

LEPTONS

Leptons (from the Greek *leptos* , meaning “small” or “light”) are a group of particles that participate in the electromagnetic and weak interactions. All leptons have spins of . Unlike hadrons, which have size and structure, leptons appear to be truly elementary point-like particles with no structure. Also unlike hadrons, the number of known leptons is small.

Mathematical formulation

METRIC TENSOR

$$\begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix}$$

MATRIX TRANSPOSITION

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

DELTA-V

In astrodynamics a Δ*v* or **delta-v** (literally "change in velocity") is a scalar which takes units of speed. It is a measure of the amount of "effort" that is needed to change from one trajectory to another by making an orbital maneuver.

Delta-v is produced by the use of propellant by reaction engines to produce a thrust that accelerates the vehicle.

DEFINITION

$$\Delta v = \int_{t_0}^{t_1} \frac{|T|}{m}\, dt$$

where

T is the instantaneous thrust

m is the instantaneous mass

In the absence of external forces:

$$= \int_{t_0}^{t_1} |a|\, dt$$

where *a* is the acceleration. When thrust is applied in a constant direction this simplifies to:

$$= |v_1 - v_0|$$

which is simply the magnitude of the change in velocity. However, this relation does not hold in the general case: if, for instance, a constant, unidirectional acceleration is reversed after $(t_1 - t_0)/2$ then the velocity difference is $t_1 - t_0 = 0$, but delta-v is the same as for the non-reversed thrust.

For rockets the 'absence of external forces' usually is taken to mean the absence of atmospheric drag as well as the absence of aerostatic back pressure on the nozzle and hence the vacuum is used for calculating the vehicle's delta-v capacity via the rocket equation, and the costs for the atmospheric losses are rolled into the delta-v budget when dealing with launches from a planetary surface.

ORBITAL MANEUVERS

Orbit maneuvers are made by firing a thruster to produce a reaction force acting on the spacecraft. The size of this force will be

$$F=V_{exh}\ \rho$$

where

V_{exh} is the velocity of the exhaust gas

ρ is the propellant flow rate to the combustion chamber

The acceleration $\dot{V}$ of the spacecraft caused by this force will be

$$\dot{V}=\frac{f}{m}=V_{exh}\frac{\rho}{m}$$

where m is the mass of the spacecraft

During the burn the mass of the spacecraft will decrease due to use of fuel, the time derivative of the mass being

$$m= -\rho$$

If now the direction of the force, i.e. the direction of the nozzle, is fixed during the burn one gets the velocity increase from the thruster force of a burn starting at time t_0 and ending at t_1 as

$$\Delta V=-\int_{t0}^{t1}V_{exh}\frac{\dot{m}}{m}dt \qquad (4)$$

Changing the integration variable from time t to the spacecraft mass m one gets

$$\Delta V=-\int_{m_0}^{m_1}V_{exh}\ \frac{dm}{m} \qquad (5)$$

Assuming V_{exh} to be a constant not depending on the amount of fuel left this relation is integrated to

$$\Delta V=V_{exh}\ \ln(\frac{m_0}{m_1})$$

(6)

which is the well known "rocket equation"

If for example 20% of the launch mass is fuel giving a constant V_{exh} of 2100 m/s (typical value for a hydrazine thruster) the capacity of the reaction control system is

$$\Delta V = 2100\ \ln(\frac{1}{0.8})$$ m/s = 469 m/s.

If V_{exh} is a non-constant function of the amount of fuel left

$$V_{exh} = V_{exh}(m)$$

the capacity of the reaction control system is computed by the integral (5)

The acceleration (2) caused by the thruster force is just an additional acceleration to be added to the other accelerations (force per unit mass) affecting the spacecraft and the orbit can easily be propagated with a numerical algorithm including also this thruster force.[2] But for many purposes, typically for studies or for maneuver optimization, they are approximated by impulsive maneuvers as illustrated in figure 1 with a ΔV as given by (4). Like this one can for example use a "patched conics" approach modeling the maneuver as a shift from one Kepler orbit to another by an instantaneous change of the velocity vector.

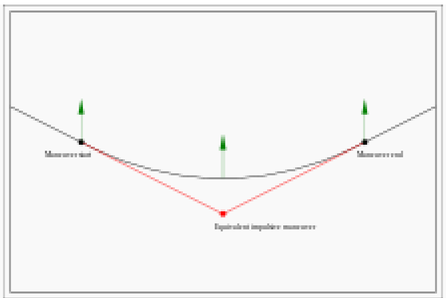


Figure 1:Approximation of a finite thrust maneuver with an impulsive change in velocity having the Delta-V given by (4)

This approximation with impulsive maneuvers is in most cases very accurate, at least when chemical propulsion is used. For low thrust systems, typically electrical propulsion systems, this approximation is less accurate. But even for geostationary spacecraft using electrical propulsion for out-of-plane control with thruster burn periods extending over several hours around the nodes this approximation is fair.

PRODUCING DELTA-V

Delta-v is typically provided by the thrust of a rocket engine, but can be created by other reaction engines. The time-rate of change of delta-v is the magnitude of the acceleration *caused by the* *engines*, i.e., the thrust per total vehicle mass. The actual acceleration vector would be found by adding thrust per mass on to the gravity vector and the vectors representing any other forces acting on the object.

The total delta-v needed is a good starting point for early design decisions since consideration of the added complexities are deferred to later times in the design process.

The rocket equation shows that the required amount of propellant dramatically increases, with increasing delta-v. Therefore in modern spacecraft propulsion systems considerable study is put into reducing the total delta-v needed for a given spaceflight, as well as designing spacecraft that are capable of producing a large delta-v.

Increasing the Delta-v provided by a propulsion system can be achieved by:

- staging
- increasing specific impulse
- improving propellant mass fraction

MULTIPLE MANEUVERS

Because the mass ratios apply to any given burn, when multiple maneuvers are performed in sequence, the mass ratios multiply.

Thus it can be shown that, provided the exhaust velocity is fixed, this means that delta-vs can be added:

When M1, M2 are the mass ratios of the maneuvers, and V1, V2 are the delta-v's of the first and second maneuvers

$$M1M2$$

$$= e^{V1/V_e} e^{V2/V_e}$$

$$= e^{(V1+V2)/V_e}$$

$$= e^{V/V_e} = M$$

Where V = V1 + V2 and M = M1 M2.

Which is just the rocket equation applied to the sum of the two maneuvers.

This is convenient since it means that delta-vs can be calculated and added and the mass ratio calculated only for the overall vehicle for the entire mission. Thus delta-v is commonly quoted rather than sequences of mass ratios.

DELTA-V BUDGETS

When designing a trajectory, delta-v budget is used as a good indicator of how much propellant will be required. Propellant usage is an exponential function of delta-v in accordance with the rocket equation, it will also depend on the exhaust velocity.

It is not possible to determine delta-v requirements from conservation of energy by considering only the total energy of the vehicle in the initial and final orbits since energy is carried away in the exhaust (see also below). For example, most spacecraft are launched in an orbit with inclination fairly near to the latitude at the launch site, to take advantage of the Earth's rotational surface speed. If it is necessary, for mission-based reasons, to put the spacecraft in an orbit of different inclination, a substantial delta-v is required, though the specific kinetic and potential energies in the final orbit and the initial orbit are equal.

When rocket thrust is applied in short bursts the other sources of acceleration may be negligible, and the magnitude of the velocity change of one burst may be simply approximated by the delta-v. The total delta-v to be applied can then simply be found by addition of each of the delta-vs needed at the discrete burns, even though between bursts the magnitude and direction of the velocity changes due to gravity, e.g. in an elliptic orbit.

For examples of calculating delta-v, see Hohmann transfer orbit, gravitational slingshot, and Interplanetary Superhighway. It is also notable that large thrust can reduce gravity drag.

Delta-v is also required to keep satellites in orbit and is expended in propulsive orbital stationkeeping maneuvers. Since the propellant load on most satellites cannot be replenished, the amount of propellant initially loaded on a satellite may well determine its useful lifetime.

BERTH EFFECT

From power considerations, it turns out that when applying delta-v in the direction of the velocity the specific orbital energy gained per unit delta-v is equal to the instantaneous speed. This is called the Oberth effect.

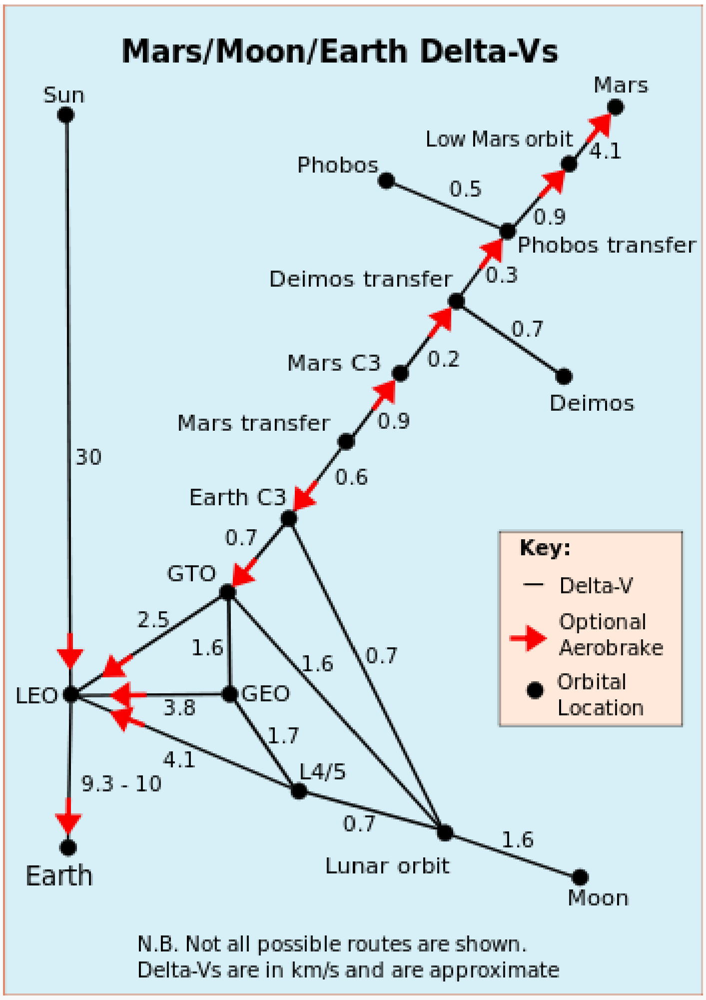
For example, a satellite in an elliptical orbit is boosted more efficiently at high speed (that is, small altitude) than at low speed (that is, high altitude).

Another example is that when a vehicle is making a pass of a planet, burning the propellant at closest approach rather than further out gives significantly higher final speed, and this is even more so when the planet is a large one with a deep gravity field, such as Jupiter.

PORKCHOP PLOT

Due to the relative positions of planets changing over time, different delta-vs are required at different launch dates. A diagram that shows the required delta-v plotted against time is sometimes called a *porkchop plot*. Such a diagram is useful since it enables calculation of a launch window, since launch should only occur when the mission is within the capabilities of the vehicle to be employed.

DELTA-VS AROUND THE SOLAR SYSTEM



*Abbreviations key: Escape orbit (**C3**), Geosynchronous orbit(**GEO**), Geostationary transfer orbit (**GTO**), Earth–Moon L_5Lagrangian point (**L5**), Low Earth orbit (**LEO**)*

Delta-vs in km/s for various orbital maneuvers using conventional rockets. Red arrows show where optional aerobraking can be performed in that particular direction, black numbers give delta-v in km/s that apply in either direction. Lower delta-v transfers than shown can often be achieved, but involve rare transfer windows or take significantly longer, see: fuzzy orbital transfers. Not all possible links are shown.

ROCKET EQUATION

Rocket mass ratios versus final velocity calculated from the rocket equation

The **Tsiolkovsky rocket equation**, or **ideal rocket equation**, describes the motion of vehicles that follow the basic principle of a rocket: a device that can apply acceleration to itself (a thrust) by expelling part of its mass with high speed and move due to the conservation of momentum. The equation relates the delta-v (the maximum change of speed of the rocket if no other external forces act) with the effective exhaust velocity and the initial and final mass of a rocket (or other reaction engine).

For any such maneuver (or journey involving a number of such maneuvers):

$$\Delta v = v_\text{e} \ln \frac{m_0}{m_1}$$

where:

m_0 is the initial total mass, including propellant,

m_1 is the final total mass,

v_e is the effective exhaust velocity ($v_\text{e} = I_\text{sp} \cdot g_0$ where I_sp is the specific impulse expressed as a time period and g_0 is Standard Gravity),

Δv is delta-v - the maximum change of speed of the vehicle (with no external forces acting),

$\ln$ refers to the natural logarithm function.

Units used for mass or velocity do not matter as long as they are consistent.

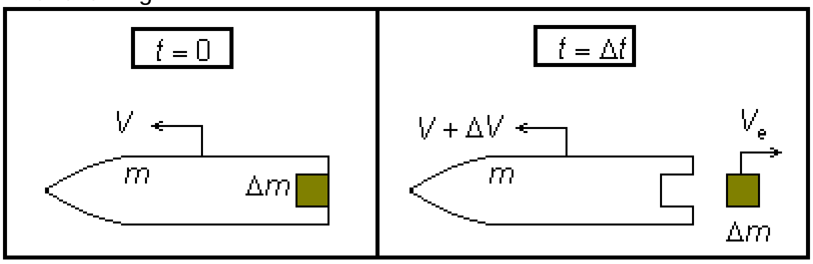
The equation is named after Konstantin Tsiolkovsky who independently derived it and published it in his 1903 work.

HISTORY

This equation was independently derived by Konstantin Tsiolkovsky towards the end of the 19th century and is widely known under his name or as the 'ideal rocket equation'. However, a recently discovered pamphlet *"A Treatise on the Motion of Rockets"* by William Moore shows that the earliest known derivation of this kind of equation was in fact at the Royal Military Academy at Woolwich in England in 1813, and was used for weapons research.

DERIVATION

Consider the following system:



In the following derivation, "the rocket" is taken to mean "the rocket and all of its unburned propellant".

Newton's second law of motion relates external forces (F_i) to the change in linear momentum of the system as follows:

$$\sum F_i = \lim_{\Delta t \to 0} \frac{P_2 - P_1}{\Delta t}$$

where P_1 is the momentum of the rocket at time *t=0*:

$$P_1 = (m + \Delta m) V$$

and P_2 is the momentum of the rocket and exhausted mass at time $t = \Delta t$:

$$P_2 = m (V + \Delta V) + \Delta m V_e$$

and where, with respect to the observer:

V is the velocity of the rocket at time *t=0*

$V + \Delta V$ is the velocity of the rocket at time $t = \Delta t$

V_e is the velocity of the mass added to the exhaust (and lost by the rocket) during time Δt

$m + \Delta m$ is the mass of the rocket at time *t=0*

m is the mass of the rocket at time $t = \Delta t$

The velocity of the exhaust V_e in the observer frame is related to the velocity of the exhaust in the rocket frame v_e by (since exhaust velocity is in the negative direction)

$$V_e = V - v_e$$

Solving yields:

$$P_2 - P_1 = m\Delta V - v_e \Delta m$$

and, using $dm = -\Delta m$, since ejecting a positive Δm results in a decrease in mass,

$$\sum F_i = m\frac{dV}{dt} + v_e\frac{dm}{dt}$$

If there are no external forces then $\sum F_i = 0$ and

$$m\frac{dV}{dt} = -v_e\frac{dm}{dt}$$

Assuming v_e is constant, this may be integrated to yield:

$$\Delta V \ = v_e \ln \frac{m_0}{m_1}$$

or equivalently

$m_1 = m_0 e^{-\Delta V \ / v_e}$ or $m_0 = m_1 e^{\Delta V \ / v_e}$ or
$m_0 - m_1 = m_1(e^{\Delta V \ / v_e} - 1)$

where m_0 is the initial total mass including propellant, m_1 the final total mass, and v_e the velocity of the rocket exhaust with respect to the rocket (the specific impulse, or, if measured in time, that multiplied by gravity-on-Earth acceleration).

The value $m_0 - m_1$ is the total mass of propellant expended, and hence:

$$M_f = 1 - \frac{m_1}{m_0} = 1 - e^{-\Delta V \ / v_e}$$

where M_f is the mass fraction (the part of the initial total mass that is spent as reaction mass).

ΔV (delta v) is the integration over time of the magnitude of the acceleration produced by using the rocket engine (what would be the actual acceleration if external forces were absent). In free space, for the case of acceleration in the direction of the velocity, this is the increase of the speed.

In the case of an acceleration in opposite direction (deceleration) it is the decrease of the speed. Of course gravity and drag also accelerate the vehicle, and they can add or subtract to the change in velocity experienced by the vehicle. Hence delta-v is not usually the actual change in speed or velocity of the vehicle.

If special relativity is taken into account, the following equation can be derived for a relativistic rocket,[4] with Δv again standing for the rocket's final velocity (after burning off all its fuel and being reduced to a rest mass of m_1) in the inertial frame of reference where the rocket started at rest (with the rest mass including fuel being m_0 initially), and c standing for the speed of light in a vacuum:

$$\frac{m_0}{m_1} = \left[\frac{1+\frac{\Delta v}{c}}{1-\frac{\Delta v}{c}}\right]^{\frac{c}{2I_{sp}}}$$

Writing $\frac{m_0}{m_1}$ as R, a little algebra allows this equation to be rearranged as

$$\frac{\Delta v}{c} = \frac{R^{\frac{2I_{sp}}{c}}-1}{R^{\frac{2I_{sp}}{c}}+1}$$

Then, using the identity $R^{\frac{2I_{sp}}{c}} = \exp\left[\frac{2I_{sp}}{c}\ln R\right]$ (here "exp" denotes the exponential function; *see also* Natural logarithm as well as the "power" identity at Logarithmic identities) and the identity $\tanh x = \frac{e^{2x}-1}{e^{2x}+1}$ (*see* Hyperbolic function), this is equivalent to

$$\Delta v = c \cdot \tanh\left(\frac{I_{sp}}{c}\ln\frac{m_0}{m_1}\right)$$

APPLICABILITY

The rocket equation captures the essentials of rocket flight physics in a single short equation. It also holds true for rocket-like reaction vehicles whenever the effective exhaust velocity is constant; and can be summed or integrated when the effective exhaust velocity varies. It does not apply to non-rocket systems, such as aerobraking, gun launches,space elevators, launch loops, tether propulsion.

Delta-v is of fundamental importance in orbital mechanics. It quantifies how difficult it is to perform a given orbital maneuver. To achieve a large delta-v, either m_0 must be huge (growing exponentially as delta-v rises), or m_1 must be tiny, or v_e must be very high, or some combination of all of these.

In practice, very-high delta-v has been achieved by a combination of 1) very large rockets (increasing m_0), 2) staging (decreasing m_1), and 3) very high exhaust velocities.

The Saturn V rocket used in the Apollo space program is an example of a large, serially staged rocket. The Space Shuttle is an example of parallel staging where all of its engines are ignited on the ground and some (the solid rocket boosters) are jettisoned to lose weight before reaching orbit.

The ion thruster is an example of a high exhaust velocity rocket. Instead of storing energy in the propellant itself as in a chemical rocket, ion and other electric rockets separate energy storage from the reaction (propellant) mass storage. Not only does this allow very large (and in principle unlimited) amounts of energy to be applied to small amounts of ejected mass to achieve very high exhaust velocities, but energy sources far more compact than chemical fuels can be used, such as nuclear reactors. In the inner solar system solar power can be used, entirely eliminating the need for a large internal primary energy storage system.

EXAMPLES

Assume an exhaust velocity of 4.5 km/s and a Δv of 9.7 km/s (Earth to LEO).

- Single stage to orbit rocket: $1 - e^{-9.7/4.5}$ = 0.884, therefore 88.4 % of the initial total mass has to be propellant. The remaining 11.6 % is for the engines, the tank, and the payload. In the case of a space shuttle, it would also include the orbiter.

- Two stage to orbit: suppose that the first stage should provide a Δv of 5.0 km/s; $1 - e^{-5.0/4.5}$ = 0.671, therefore 67.1% of the initial total mass has to be propellant to the first stage. The remaining mass is 32.9 %. After disposing of the first stage, a mass remains equal to this 32.9 %, minus the mass of the tank and engines of the first stage. Assume that this is 8 % of the initial total mass, then 24.9 % remains. The second stage should provide a Δv of 4.7 km/s; $1 - e^{-4.7/4.5}$ = 0.648, therefore 64.8% of the remaining mass has to be propellant, which is 16.2 %, and 8.7 % remains for the tank and engines of the second stage, the payload, and in the case of a space shuttle, also the orbiter. Thus together 16.7 % is available for all engines, the tanks, the payload, and the possible orbiter.

STAGES

In the case of sequentially thrusting rocket stages, the equation applies for each stage, where for each stage the final mass in the equation is the total mass of the rocket after discarding the previous stage, and the initial mass in the equation is the total mass of the rocket just before discarding the stage concerned. For each stage the specific impulse may be different.

For example, if 80% of the mass of a rocket is the fuel of the first stage, and 10% is the dry mass of the first stage, and 10% is the remaining rocket, then

$$\begin{aligned}\Delta v &= v_{\rm e} \ln \frac{100}{100-80} \\ &= v_{\rm e} \ln 5 \\ &= 1.61 v_{\rm e}.\end{aligned}$$

With three similar, subsequently smaller stages with the same v_e for each stage, we have

$$\Delta v = 3 v_{\rm e} \ln 5 = 4.83 v_{\rm e}$$

and the payload is 10%*10%*10% = 0.1% of the initial mass.

A comparable SSTO rocket, also with a 0.1% payload, could have a mass of 11% for fuel tanks and engines, and 88.9% for fuel. This would give

$$\Delta v = v_{\rm e} \ln(100/11.1) = 2.20 v_{\rm e}.$$

If the motor of a new stage is ignited before the previous stage has been discarded and the simultaneously working motors have a different specific impulse (as is often the case with solid rocket boosters and a liquid-fuel stage), the situation is more complicated.

COMMON MISCONCEPTIONS

Because this is a variable-mass system, Newton's second law of motion cannot directly be applied because it is valid for constant mass systems only. It can cause confusion that the Tsiolkovsky rocket equation is similar to the relativistic force equation $F = dp/dt = m\ dv/dt + v\ dm/dt$. Using this formula with $m(t)$ as the varying mass of the rocket is mathematically equivalent to the derived Tsiolkovsky rocket equation, but this derivation is not correct.

A simple counter example is to consider a rocket travelling with a constant velocity v with two maneuvering thrusters pointing out on either side, with both firing such that their forces cancel each other out. In such a case the rocket would be losing mass and an incorrect application of $F = dp/dt$ would result in a non-zero but non-accelerating force, leading to nonsensical answers.

Any system with a non-constant mass must be treated as a variable-mass system.

VIS VIVA

EQUATION

In astrodynamics, the ***vis viva* equation**, also referred to as **orbital energy conservation equation**, is one of the fundamental and useful equations that govern the motion of orbiting bodies. It is the direct result of the law of conservation of energy, which requires that the sum of kinetic and potential energy be constant at all points along the orbit.

Vis viva (Latin for "live force") is a term from the history of mechanics, and it survives in this sole context. It represents the principle that the difference between the aggregate work of the accelerating forces of a system and that of the retarding forces is equal to one half the *vis viva* accumulated or lost in the system while the work is being done.

For any Kepler orbit (elliptic, parabolic, hyperbolic or radial), the *vis viva* equation[1] is as follows:

$$v^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right)$$

where:

- v is the relative speed of the two bodies
- r is the distance between the two bodies
- a is the semi-major axis (a>0 for ellipses, $a = \infty$ or $\frac{1}{a}$=0 for parabolas, and a<0 for hyperbolas)
- G is the gravitational constant
- M, m are the masses of the two bodies

DERIVATION

The total orbital energy is the sum of the shared potential energy, the kinetic energy of body M, and the kinetic energy of body m

$$E = \frac{-GMm}{r} + \frac{Mv_M^2}{2} + \frac{mv_m^2}{2}$$

- v_M is the speed of body M relative to the inertial center of mass of the two bodies.
- v_m is the speed of body m relative to the inertial center of mass of the two bodies.

The orbital energy can also be calculated using only relative quantities

$$E = \frac{-GMm}{r} + \mu\frac{v^2}{2}$$

- v is the relative speed of the two bodies
- $\mu = \frac{Mm}{M+m}$ is the reduced mass

For elliptic and circular orbits, the total energy is given more concisely by

$$E = \frac{-GMm}{2a},$$

- a is the semi-major axis.

Dividing the total energy by the reduced mass gives the *vis-viva* energy, more commonly known in modern times as the specific orbital energy

$$\epsilon = \frac{v^2}{2} - \frac{G(M+m)}{r}$$

For elliptic and circular orbits

$$\epsilon = \frac{-G(M+m)}{2a}$$

Equating the two previous expressions and solving for v yields the *vis viva* equation:

$$v^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right).$$

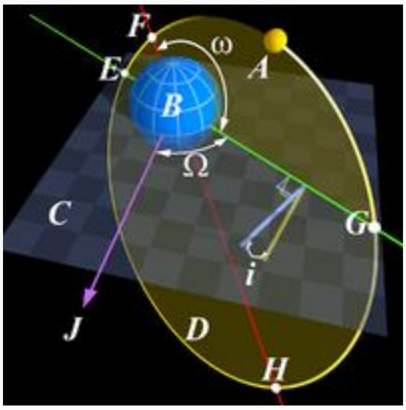
PRACTICAL APPLICATIONS

Given the total mass and the scalars *r* and *v* at a single point of the orbit, one can compute *r* and *v* at any other point in the orbit.[2]

Given the total mass and the scalars *r* and *v* at a single point of the orbit, one can compute the specific orbital energy ϵ, allowing an object orbiting a larger object to be classified as having not enough energy to remain in orbit, hence being "suborbital" (a ballistic missile, for example), having enough energy to be "orbital", but without the possibility to complete a full orbit anyway because it eventually collides with the other body, or having enough energy to come from and/or go to infinity (as a meteor, for example).

Orbital Mechanics

Astrodynamics



Orbital mechanics

FOCI

The foci of the ellipse are two special points F_1 and F_2 on the ellipse's major axis and are equidistant from the center point. The sum of the distances from any point P on the ellipse to those two foci is constant and equal to the major axis ($PF_1 + PF_2 = 2a$). Each of these two points is called a **focus** of the ellipse.

GRAVITY ASSISTS

OBERTH EFFECT

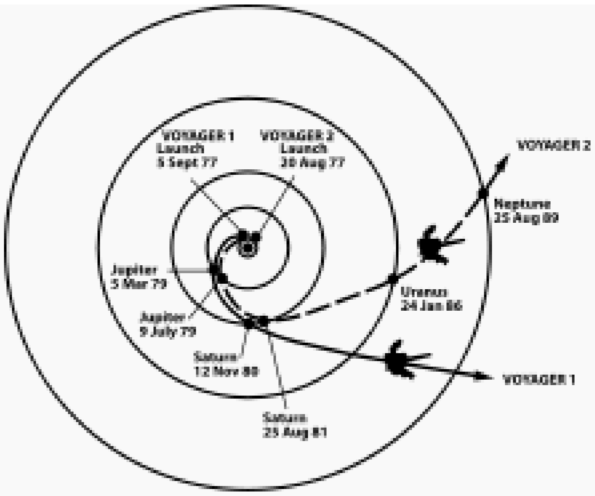
In astronautics, the **Oberth effect** is where the use of a rocket engine when travelling at high speed generates much more useful energy than one at low speed. Oberth effect occurs because the propellant has more usable energy (due to its kinetic energy on top of its chemical potential energy) and it turns out that the vehicle is able to employ this kinetic energy to generate more mechanical power. It is named after Hermann Oberth, the Austro-Hungarian-born, German physicist and a founder of modern rocketry, who apparently first described the effect.

Oberth effect is used in a **powered flyby** or **Oberth maneuver** where the application of an impulse, typically from the use of a rocket engine, close to a gravitational body (where the gravity potential is low, and the speed is high) can give much more change in kinetic energy and final speed (i.e. higher specific energy) than the same impulse applied further from the body for the same initial orbit. For the Oberth effect to be most effective, the vehicle must be able to generate as much impulse as possible at the lowest possible altitude; thus the Oberth effect is often far less useful for low-thrust reaction engines such as ion drives, which have a low propellant flow rate.

Oberth effect also can be used to understand the behaviour of multi-stage rockets; the upper stage can generate much more usable kinetic energy than might be expected from simply considering the chemical energy of the propellants it carries.

Historically, a lack of understanding of this effect led early investigators to conclude that interplanetary travel would require completely impractical amounts of propellant, as without it, enormous amounts of energy are needed.

GRAVITATIONAL ASSIST



The trajectories that enabled NASA's twin Voyager spacecraft to tour the four gas giant planets and achieve velocity to escape our solar system

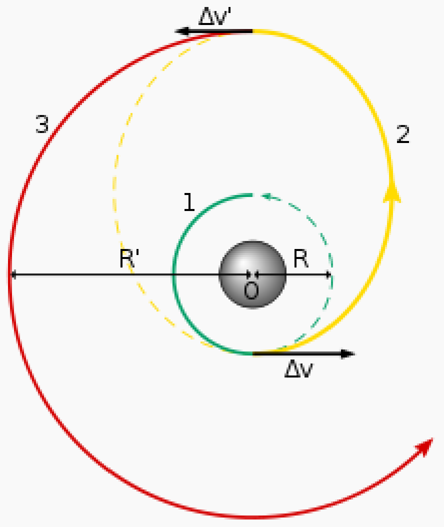
In orbital mechanics and aerospace engineering, a **gravitational slingshot**, **gravity assist maneuver**, or **swing-by** is the use of the relative movement and gravity of a planet or other celestial body to alter the path and speed of a spacecraft, typically in order to save propellant, time, and expense. Gravity assistance can be used to accelerate, decelerate and/or re-direct the path of a spacecraft.

The "assist" is provided by the motion (orbital angular momentum) of the gravitating body as it pulls on the spacecraft.[2] The technique was first proposed as a mid-course manoeuvre in 1961, and used by interplanetary probes from *Mariner 10* onwards, including the two *Voyager* probes' notable fly-bys of Jupiter and Saturn.

TRANSFER ORBITS

Orbit insertion is a general term for a maneuver that is more than a small correction. It may be used for a maneuver to change a transfer orbit or an ascent orbit into a stable one, but also to change a stable orbit into a descent: *descent orbit insertion*. Also the term **orbit injection** is used, especially for changing a stable orbit into a transfer orbit, e.g. trans-lunar injection (TLI), *trans-Mars injection* (TMI) and trans-Earth injection (TEI).

HOHMANN TRANSFER



Hohmann Transfer Orbit

In orbital mechanics, the **Hohmann transfer orbit** is an elliptical orbit used to transfer between two circular orbits of different altitudes, in the same plane.

The **orbital maneuver** to perform the Hohmann transfer uses two engine impulses which move a spacecraft onto and off the transfer orbit. This maneuver was named after Walter Hohmann, the German scientist who published a description of it in his 1925 book *Die Erreichbarkeit der Himmelskörper* (*The Accessibility of Celestial Bodies*) Hohmann was influenced in part by the German science fiction author Kurd Laßwitz and his 1897 book *Two Planets*.

BI-ELLIPTIC TRANSFER

In astronautics and aerospace engineering, the **bi-elliptic transfer** is an **orbital maneuver** that moves a spacecraft from one orbitto another and may, in certain situations, require less delta-v than a Hohmann transfer maneuver.

The bi-elliptic transfer consists of two half elliptic orbits. From the initial orbit, a delta-v is applied boosting the spacecraft into the first transfer orbit with an apoapsis at some point r_b away from the central body. At this point, a second delta-v is applied sending the spacecraft into the second elliptical orbit with periapsis at the radius of the final desired orbit, where a third delta-v is performed, injecting the spacecraft into the desired orbit.

While they require one more engine burn than a Hohmann transfer and generally requires a greater travel time, some bi-elliptic transfers require a lower amount of total delta-v than a Hohmann transfer when the ratio of final to initial semi-major axis is 11.94 or greater, depending on the intermediate semi-major axis chosen.

The idea of the bi-elliptical transfer trajectory was first published by Ary Sternfeld in 1934.[5]

LOW ENERGY TRANSFER

Main article: low energy transfer

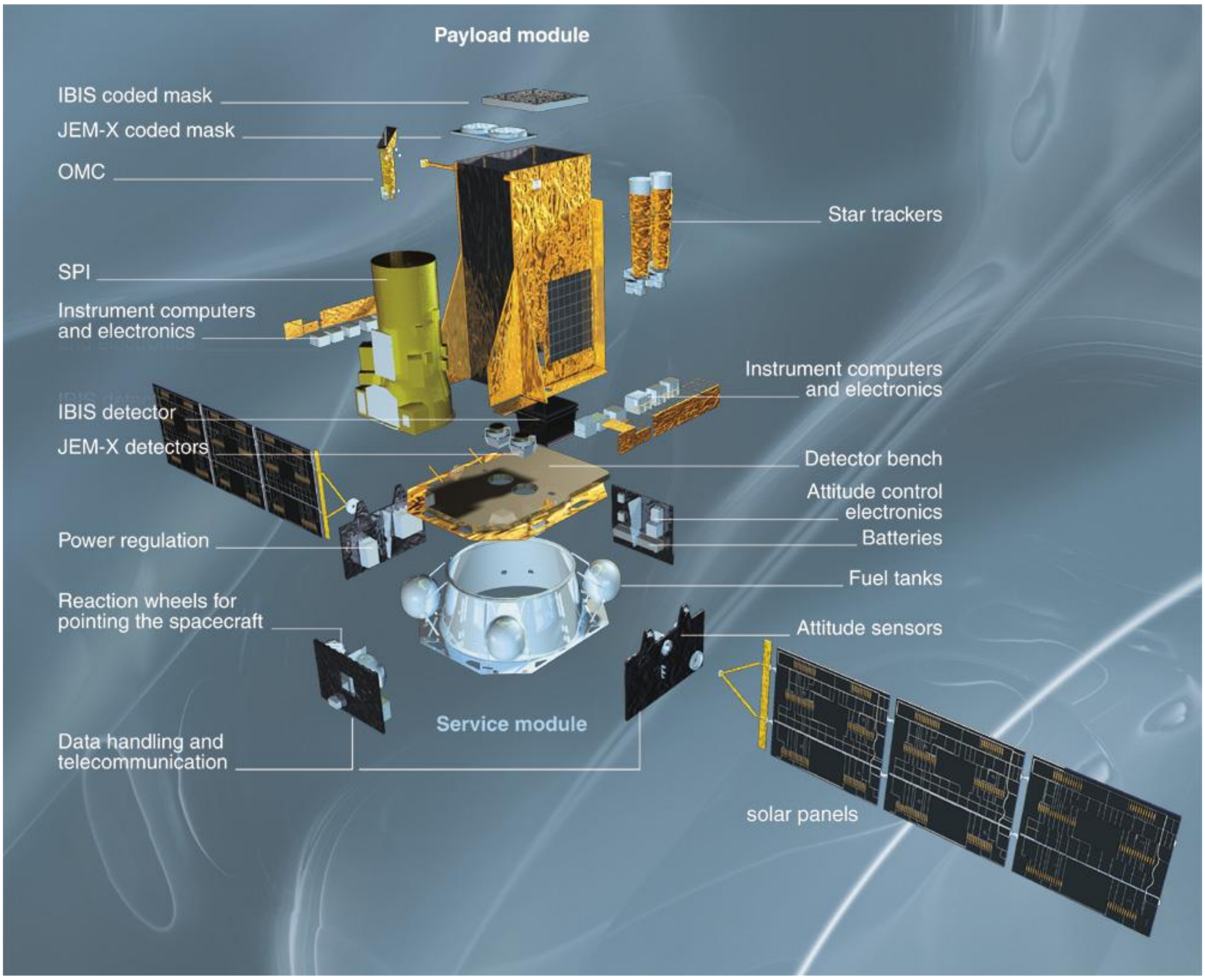
A **low energy transfer**, or low energy trajectory, is a route in space which allows spacecraft to change orbits using very little fuel. [6][7] These routes work in the Earth-Moonsystem and also in other systems, such as traveling between the satellites of Jupiter. The drawback of such trajectories is that they take much longer to complete than higher energy (more fuel) transfers such as Hohmann transfer orbits.

Low energy transfer are also known as weak stability boundary trajectories, or ballistic capture trajectories.

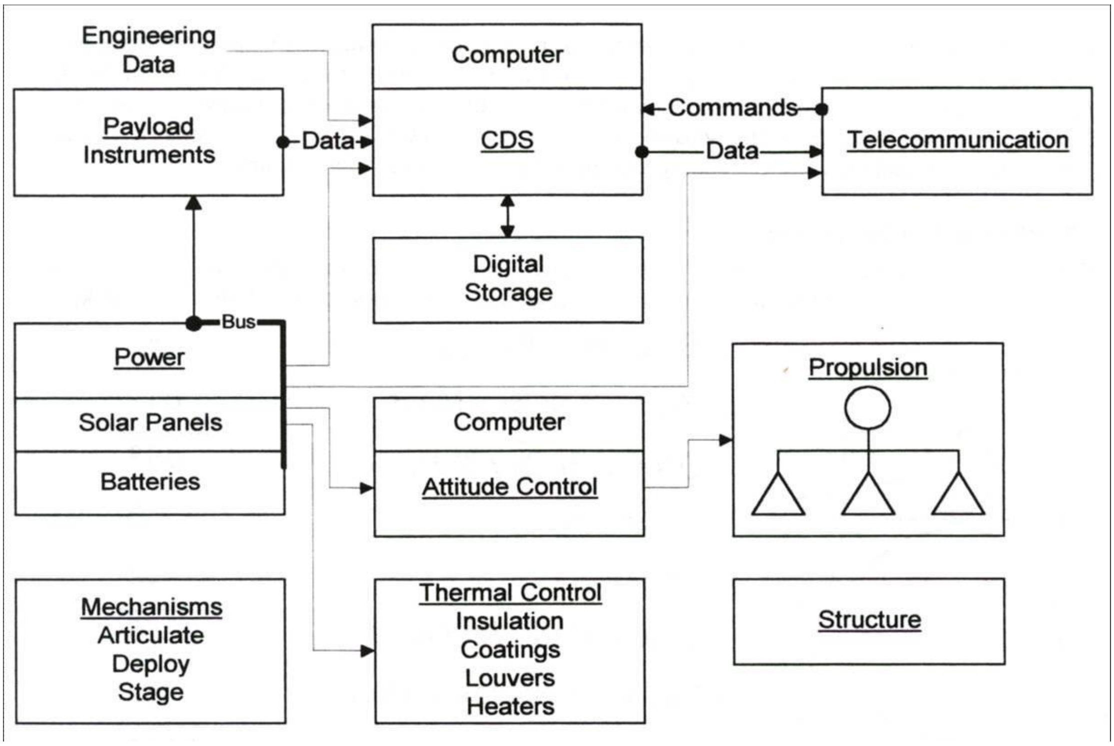
Low energy transfers follow special pathways in space, sometimes referred to as the Interplanetary Transport Network. Following these pathways allows for long distances to be traversed for little expenditure of delta-v.

Spacecraft Engineering

SUBSYSTEMS



- Altitude control
- Power
- Propulsion
- Structure
- Telemetry
- Thermal
- Timing control
- Instrument
- Other



Orbiting anomaly

- Subsystem
- Component
- Assembly
- Part

Decision making

- Negligible 5%
- Minor -30%
- Substansial -60%
- Major -95%
- Catastrophic -100%

ELEMENTS OF SOAR REPORT

- Anomaly effect
- Failure category
- Type of anomaly

MOND

STANDS FOR MODIFIED NEWTONIAN DYNAMICS.
IT'S MAIN APPLICATION IS TO MODEL THE UNIVERSE ON THE LARGEST SCALE - GALAXY DYNAMICS.

MOND USES SIMPLE STATISICAL METHODS TO PREDICT THE MOTION OF GALAXYS. ROUGHLY SPEAKING, THE METHODS USED TO CONSTRUCT MOND ARE SIMILAR TO STANDARD LINEAR REGRESSION TECHNIQUES FOUND IN COLLEGE LEVEL STATISTICS BOOK.

MOND IS VERY SPARSE ON THEORETICAL FRAMEWORK, UNLIKE THEORIES SUCH AS GR. IT'S MAIN ASSERTION IS THAT " IN THE CASE OF INTERGALACTIC SCALE, GRAVITATIONAL FORCE DOES NOT VARY PROPORTIONAL TO THE INVERSE SQUARE OF THE DISTANCE (1/R^2), BUT TEND TOWARDS THE INVERSE (1/R). " . THERE IS NO OFFICIAL REASON ATTACHED TO MOND AS TO WHY NEWTON’S INVERSE SQUARE LAW BREAKS DOWN AT THIS SCALE.

MOND WAS CREATED BECAUSE IT BECAME APPARENT TO PHYSICISTS THAT EXISTING MODELS WERE NOT PRODUCING RESULTS CONSISTENT WITH OBSERVATIONS. I.E. GR AND DARK MATTER MODELS.

MOND IS BY FAR THE BEST APPROXIMATION FOR INTER- GALACTIC DYNAMICS.

Predicted rotation curve

Far away from the center of a galaxy, the gravitational acceleration a star undergoes is predicted by MOND to be roughly:

$$\mu\left(\frac{a}{a_0}\right)a = \frac{GM}{r^2}$$

with G the gravitation constant, M the mass of the galaxy, and r the distance between the center and the star.

The exact form of μ is unspecified, only its behavior when the argument a/a_0 is small or large. As Mordehai Milgrom proved in his original paper, the form of μ does not change most of the consequences of the theory, such as the flattening of the rotation curve.

Assuming that, at this large distance r, a is smaller than a_0, $\mu\left(\frac{a}{a_0}\right) = \frac{a}{a_0}$

This gives:

$$\frac{GM}{r^2} = \frac{a^2}{a_0}$$

Therefore:

$$a = \frac{\sqrt{GMa_0}}{r}$$

Since the equation that relates the velocity to the acceleration for a circular orbit is $a = \frac{v^2}{r}$, one has:

$$a = \frac{v^2}{r} = \frac{\sqrt{GMa_0}}{r}$$

and therefore:

$$v = \sqrt[4]{GMa_0}$$

SCHRÖDINGER EQUATION

In quantum mechanics, the **Schrödinger equation** is a partial differential equation that describes how the quantum state of a physical system changes with time. It was formulated in late 1925, and published in 1926, by the Austrian physicist Erwin Schrödinger.

In classical mechanics, the equation of motion is Newton's second law, and equivalent formulations are the Euler–Lagrange equationsand Hamilton's equations. In all these formulations, they are used to solve for the motion of a mechanical system, and mathematically predict what the system will do at any time beyond the initial settings and configuration of the system.

In quantum mechanics, the analogue of Newton's law is Schrödinger's equation for a quantum system, usually atoms, molecules, and subatomic particles; free, bound, or localized. It is not a simple algebraic equation, but (in general) a linear partial differential equation. The differential equation describes the wavefunction of the system, also called the quantum state or state vector.

In the standard interpretation of quantum mechanics, the wavefunction is the most complete description that can be given to a physical system. Solutions to Schrödinger's equation describe not only molecular, atomic, and subatomic systems, but also macroscopic systems, possibly even the whole universe.[1]

Like Newton's Second law, the Schrödinger equation can be mathematically transformed into other formulations such as Werner Heisenberg's matrix mechanics, and Richard Feynman's path integral formulation. Also like Newton's Second law, the Schrödinger equation describes time in a way that is inconvenient for relativistic theories, a problem that is not as severe in matrix mechanics and completely absent in the path integral formulation.

EQUATION

TIME-DEPENDENT EQUATION

The form of the Schrödinger equation depends on the physical situation (see below for special cases). The most general form is the time-dependent Schrödinger equation, which gives a description of a system evolving with time

Time-dependent Schrödinger equation *(general)*

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

where *Ψ* is the wave function of the quantum system, *i* is the imaginary unit, *ħ* is the reduced Planck constant, and $\hat{H}$ is the Hamiltonian operator, which characterizes the total energy of any given wavefunction and takes different forms depending on the situation.

A wave function which satisfies the non-relativistic Schrödinger equation with *V*=0. In other words, this corresponds to a particle traveling freely through empty space. The real part of the wave function is plotted here.

The most famous example is the non-relativistic Schrödinger equation for a single particle moving in an electric field (but not amagnetic field):

Time-dependent Schrödinger equation *(single non-relativistic particle)*

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

where *m* is the particle's mass, *V* is its potential energy, ∇^2 is the Laplacian, and *Ψ* is the wavefunction (more precisely, in this context, it is called the "position-space wavefunction"). In plain language, it means "total energy equals kinetic energy plus potential energy", but the terms take unfamiliar forms for reasons explained below.

Given the particular differential operators involved, this is a linear partial differential equation. It is also a diffusion equation.

The term *"Schrödinger equation"* can refer to both the general equation (first box above), or the specific nonrelativistic version (second box above and variations thereof). The general equation is indeed quite general, used throughout quantum mechanics, for everything from the Dirac equation to quantum field theory, by plugging in various complicated expressions for the Hamiltonian. The specific nonrelativistic version is a simplified approximation to reality, which is quite accurate in many situations, but very inaccurate in others (see relativistic quantum mechanics).

To apply the Schrödinger equation, the Hamiltonian operator is set up for the system, accounting for the kinetic and potential energy of the particles constituting the system, then inserted into the Schrödinger equation. The resulting partial differential equation is solved for the wave function, which contains information about the system.

CURVATURE(SOURCE:WIKIPEDIA)

The metric *g* completely determines the curvature of spacetime. According to the fundamental theorem of Riemannian geometry, there is a unique connection ∇ on any semi-Riemannian manifold that is compatible with the metric and torsion-free. This connection is called the Levi-Civita connection. The Christoffel symbols of this connection are given in terms of partial derivatives of the metric in local coordinates x^μ by the formula

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\frac{\partial g_{\rho\mu}}{\partial x^\nu} + \frac{\partial g_{\rho\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right).$$

The curvature of spacetime is then given by the Riemann curvature tensor which is defined in terms of the Levi-Civita connection ∇. In local coordinates this tensor is given by:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}.$$

The curvature is then expressible purely in terms of the metric g and its derivatives.

EINSTEIN'S EQUATIONS

One of the core ideas of general relativity is that the metric (and the associated geometry of spacetime) is determined by the matter and energy content of spacetime.Einstein's field equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

where

$$R_{\nu\rho} \stackrel{\text{def}}{=} R^{\mu}{}_{\nu\mu\rho}$$

relate the metric (and the associated curvature tensors) to the stress-energy tensor $T_{\mu\nu}$. This tensor equation is a complicated set of nonlinear partial differential equations for the metric components. Exact solutions of Einstein's field equations are very difficult to find.

METRIC TENSOR

Flat :

ds^2=-dt^2+dx^2+dy^2+dz^2

{-,+,+,+}

Spherical :

ds^2=-dt^2+dr^2+r^{2}dΩ^2

{-,+,+}

SCHWARZSCHILD METRIC

Besides the flat space metric the most important metric in general relativity is the Schwarzschild metric which can be given in one set of local coordinates by

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

where, again, $d\Omega^2$ is the standard metric on the 2-sphere. Here *G* is the gravitation constant and *M* is a constant with the dimensions of mass. Its derivation can be foundhere. The Schwarzschild metric approaches the Minkowski metric as *M* approaches zero (except at the origin where it is undefined). Similarly, when *r* goes to infinity, the Schwarzschild metric approaches the Minkowski metric.

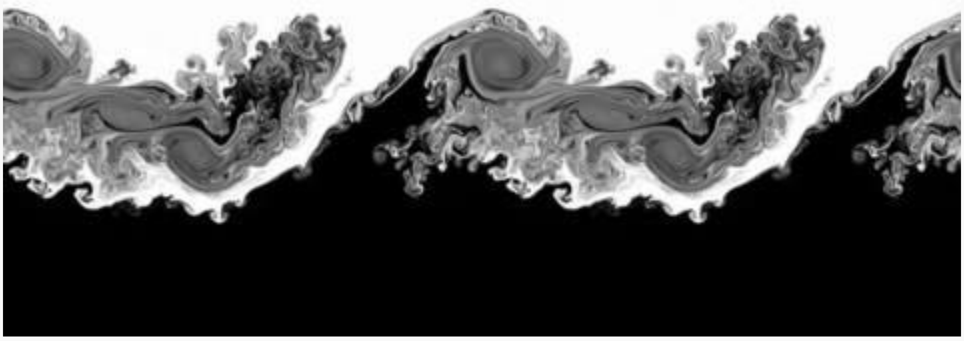
OTHER METRICS

Other notable metrics are Eddington–Finkelstein coordinates, Friedmann–Lemaître–Robertson–Walker metric, Gullstrand–Painlevé coordinates, Isotropic coordinates, Kerr metric, Kerr–Newman metric, Kruskal–Szekeres coordinates, Lemaitre metric, Reissner–Nordström metric, Rindler coordinates. Some of them are without the event horizon or can be without the gravitational singularity.

TAYLOR-GOLDSTEIN EQUATION AND STABILITY

Taylor-Goldstein equation (TGE) governs the stability of a shear-flow of an inviscid fluid of variable density. It is investigated here from a rigorous geometrical point of view using a canonical class of its transformations. Rayleigh's point of inflection criterion and Fjortoft's condition of instability of a homogenous shear-flow have been generalized here so that only the profile carrying the point of inflection is modified by the variation of density. This fulfils a persistent expectation in the literature. A pair of bounds exists such that in any unstable flow the flow-curvature (a function of flow-layers) exceeds the upper bound at some flow-layer and falls below the lower bound at a higher layer. This is the main result proved here. Bounds are obtained on the growth rate and the wave numbers of unstable modes, in fulfillment of longstanding predictions of Howard. A result of Drazin and Howard on the boundedness of the wave numbers is generalized to TGE. The results above hold if the local Richardson number does not exceed 1/4 anywhere in the flow, otherwise a weakening of the conditions necessary for instability is seen. Conditions for the propagation of neutrally stable waves and bounds on the phase speeds of destabilizing waves are obtained. It is also shown that the set of complex wave velocities of normal modes of an arbitrary flow is bounded. Fundamental solutions of TGE are obtained and their smoothness is examined. Finally sufficient conditions for instability are suggested.

KELVIN–HELMHOLTZ INSTABILITY



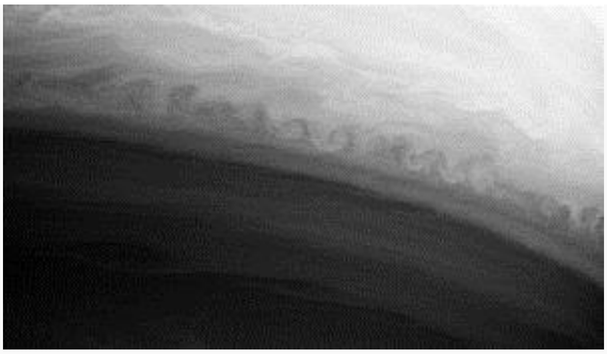
Numerical simulation of a temporal Kelvin–Helmholtz instability

The **Kelvin–Helmholtz instability** (after Lord Kelvin and Hermann von Helmholtz) can occur when there is velocity shear in a single continuous fluid, or where there is a velocity difference across the interface between two fluids. An example is wind blowing over water: The instability manifests in waves on the water surface. More generally, clouds, the ocean, Saturn's bands, Jupiter's Red Spot, and the sun's corona show this instability.

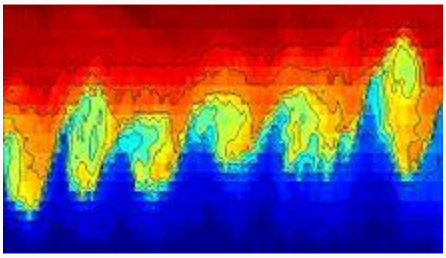
The theory predicts the onset of instability and transition to turbulent flow in fluids of different densities moving at various speeds. Helmholtz studied the dynamics of two fluids of different densities when a small disturbance, such as a wave, was introduced at the boundary connecting the fluids.

A KH instability rendered visible by clouds over Mount Duval in Australia

For some short enough wavelengths, if surface tension is ignored, two fluids in parallel motion with different velocities and densities yield an interface that is unstable for all speeds. Surface tension stabilises the short wavelength instability however, and theory predicts stability until a velocity threshold is reached. The theory with surface tension included broadly predicts the onset of wave formation in the important case of wind over water.



A KH instability on the planet Saturn, formed at the interaction of two bands of the planet's atmosphere



Kelvin-Helmholtz billows 500m deep in the Atlantic Ocean

In gravity, for a continuously varying distribution of density and velocity (with the lighter layers uppermost, so that the fluid is RT-stable), the dynamics of the KH instability is described by the Taylor–Goldstein equation and its onset is given by a Richardson number, Ri. Typically the layer is unstable for Ri<0.25. These effects are common in cloud layers. The study of this instability is applicable in plasma physics, for example in inertial confinement fusion and the plasma–beryllium interface.

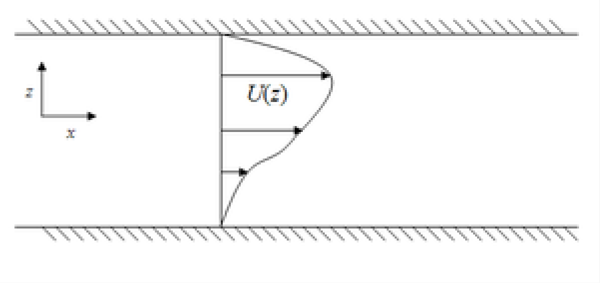
Numerically, the KH instability is simulated in a temporal or a spatial approach. In the temporal approach, experimenters consider the flow in a periodic (cyclic) box "moving" at mean speed (absolute instability). In the spatial approach, experimenters simulate a lab experiment with natural inlet and outlet conditions

TAYLOR–GOLDSTEIN EQUATION

The **Taylor–Goldstein equation** is an ordinary differential equation used in the fields of geophysical fluid dynamics, and more generally in fluid dynamics, in presence of quasi-2D flows.[1] It describes the dynamics of the Kelvin–Helmholtz instability, subject tobuoyancy forces (e.g. gravity), for stably-stratified fluids in the dissipation-less limit. Or, more generally, the dynamics of internal waves in the presence of a (continuous) density stratification and shear flow. The Taylor–Goldstein equation is derived from the 2DEuler equations, using the Boussinesq approximation.[2]

The equation is named after G.I. Taylor and S. Goldstein, who derived the equation independently from each other in 1931. The third independent derivation, also in 1931, was made by B. Haurwitz.[2]

FORMULATION



A schematic diagram of the base state of the system. The flow under investigation represents a small perturbation away from this state. While the base state is parallel, the perturbation velocity has components in both directions.

The equation is derived by solving a linearized version of the Navier–Stokes equation, in presence of gravity g and a mean density gradient (with gradient-length L_ρ), for the perturbation velocity field

$$\mathbf{u} = [U(z) + u'(x,z,t), 0, w'(x,z,t)],$$

where $(U(z),0,0)$ is the unperturbed or basic flow. The perturbation velocity has the wave-like solution $\mathbf{u}' \propto \exp(i\alpha(x-ct))$ (real partunderstood). Using this knowledge, and the streamfunction representation $u_x' = -i\alpha\tilde{\phi}, u_z' = d\tilde{\phi}/dz$ for the flow, the following dimensional form of the Taylor–Goldstein equation is obtained:

$$(U-c)^2\left(\frac{d^2\tilde{\phi}}{dz^2} - \alpha^2\tilde{\phi}\right) + \left[N^2 - (U-c)\frac{d^2U}{dz^2}\right]\tilde{\phi} = 0,$$

where $N = \sqrt{\dfrac{g}{L_\rho}}$ denotes the Brunt–Väisälä frequency. The eigenvalue parameter of the problem is c. If the imaginary part of thewave speed c is positive, then the flow is unstable, and the small perturbation introduced to the system is amplified in time.

Note that a purely-imaginary Brunt–Väisälä frequency N results in a flow which is always unstable. This instability is known as theRayleigh–Taylor instability.

RAYLEIGH–TAYLOR INSTABILITY

RT fingers evident in the Crab Nebula

The **Rayleigh–Taylor instability**, or **RT instability** (after Lord Rayleigh andG. I. Taylor), is an instability of an interface between two fluids of differentdensities, which occurs when the lighter fluid is pushing the heavier fluid. This is the case with an interstellar cloud and shock system. The equivalent situation occurs when gravity is acting on two fluids of different density – with the dense fluid above a fluid of lesser density – such as water balancing on light oil.

Consider two completely plane-parallel layers of immiscible fluid, the more dense on top of the less dense one and both subject to the Earth's gravity. The equilibrium here is unstable to certain perturbations or disturbances. An unstable disturbance will grow and lead to a release of potential energy, as the more dense material moves down under the (effective) gravitational field, and the less dense material is displaced upwards. This was the set-up as studied by Lord Rayleigh. The important insight by G. I. Taylor was, that he realised this situation is equivalent to the situation when the fluids areaccelerated, with the less dense fluid accelerating into the more dense fluid.[2]This occurs deep underwater on the surface of the expanding bubble surrounding a nuclear explosion.

As the instability develops, outward-moving irregularities ('dimples') are quickly magnified into sets of inter-penetrating **Rayleigh–Taylor fingers**. Therefore the Rayleigh–Taylor instability is sometimes qualified to be a fingering instability.[4] The upward-moving, less dense material is shaped like *mushroom caps*.

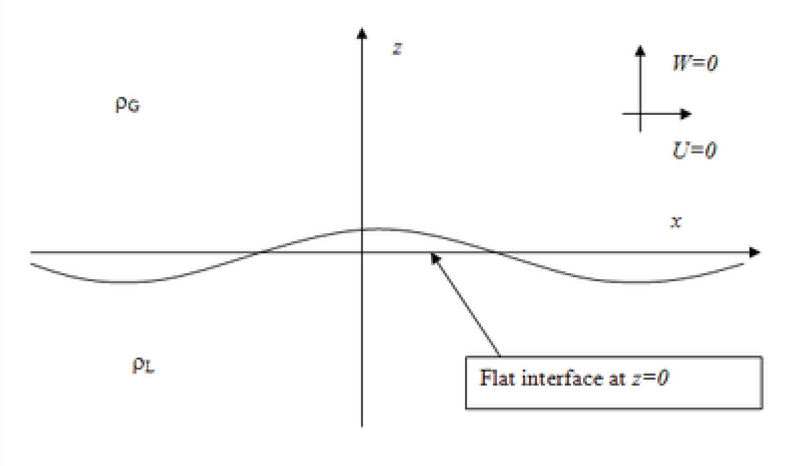
This process is evident not only in many terrestrial examples, from salt domes to weather inversions, but also in astrophysics andelectrohydrodynamics. RT fingers are especially obvious in the Crab Nebula, in which the expanding pulsar wind nebula powered by the Crab pulsar is sweeping up ejected material from the supernova explosion 1000 years ago.

Note that the RT instability is not to be confused with the Plateau-Rayleigh instability (also known as Rayleigh instability) of a liquid jet. This instability, sometimes called the hosepipe (or firehose) instability, occurs due to surface tension, which acts to break a cylindrical jet into a stream of droplets having the same volume but lower surface area.

DEMONSTRATING THE INSTABILITY IN THE KITCHEN

The Rayleigh-Taylor instability can be demonstrated using common household items. The experiment consists of adding three tablespoons of molasses to a large, heat-resistant glass, and filling up with milk. The glass has to be transparent so that the instability can be seen. The glass is then put in a microwave oven, and heated on maximum power until the instability occurs. This happens before the milk boils. When the molasses heats sufficiently, it becomes less dense than the milk above it, and the instability occurs. Shortly after this, the molasses and milk mix together, forming a light brown liquid.

LINEAR STABILITY ANALYSIS



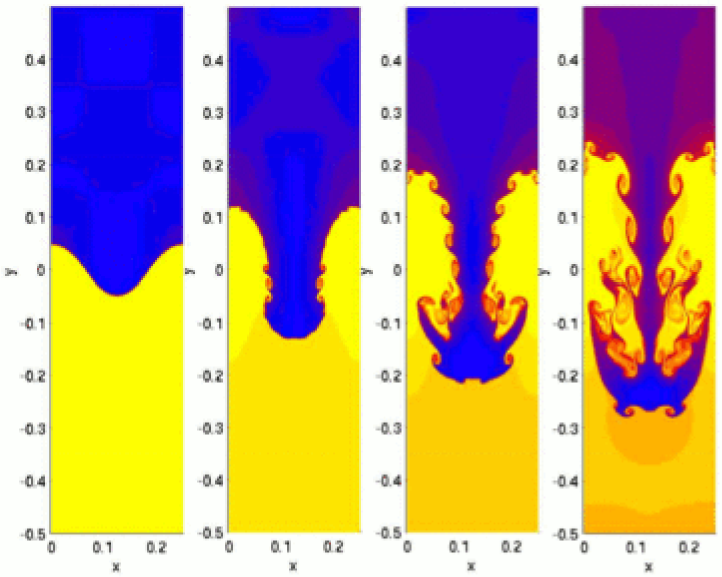
Base state of the Rayleigh–Taylor instability. Gravity points downwards.

The inviscid two-dimensional Rayleigh–Taylor (RT) instability provides an excellent springboard into the mathematical study of stability because of the exceptionally simple nature of the base state.[8] This is the equilibrium state that exists before any perturbation is added to the system, and is described by the mean velocity field $U(x,z)=W(x,z)=0,$ where thegravitational field is $\mathbf{g}=-g\hat{\mathbf{z}}.$ An interface at $z=0$separates the fluids of densities ρ_G in the upper region, and ρ_L in the lower region. In this section it is shown that when the heavy fluid sits on top, the growth of a small perturbation at the interface is exponential, and takes place at the rate[2]

$$\exp(\gamma t)\;, \qquad \text{with} \quad \gamma=\sqrt{\mathcal{A}g\alpha} \quad \text{and} \quad \mathcal{A}=\frac{\rho_{\text{heavy}}-\rho_{\text{light}}}{\rho_{\text{heavy}}+\rho_{\text{light}}},$$

where γ is the temporal growth rate, α is the spatial wavenumber and $\mathcal{A}$ is the Atwood number.

Details of the linear stability analysis



Hydrodynamics simulation of a single "finger" of the Rayleigh–Taylor instability[10] Note the formation of Kelvin–Helmholtz instabilities, in the second and later snapshots shown (starting initially around the level $y = 0$), as well as the formation of a "mushroom cap" at a later stage in the third and fourth frame in the sequence.

The time evolution of the free interface elevation $z = \eta(x, t),$ initially at $\eta(x, 0) = \Re \left\{ B \, \exp \left(i \alpha x \right) \right\},$ is given by:

$$\eta = \Re \left\{ B \, \exp \left(\sqrt{\mathcal{A} g \alpha} \, t \right) \, \exp \left(i \alpha x \right) \right\}$$

which grows exponentially in time. Here *B* is the amplitudeof the initial perturbation, and $\Re \left\{ \cdot \right\}$ denotes the real partof the complex valued expression between brackets.

In general, the condition for linear instability is that the imaginary part of the "wave speed" *c* be positive. Finally, restoring the surface tension makes c^2 less negative and is therefore stabilizing. Indeed, there is a range of short waves for which the surface tension stabilizes the system and prevents the instability forming.

LATE-TIME BEHAVIOUR

The analysis of the previous section breaks down when the amplitude of the perturbation is large. The growth then becomes non-linear as the spikes and bubbles of the instability tangle and roll up into vortices. Then, as in the figure, numerical simulation of the full problem is required to describe the system.

NAVIER–STOKES EQUATIONS

In physics, the **Navier–Stokes equations**, named after Claude-Louis Navier and George Gabriel Stokes, describe the motion of fluid substances. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term.

The equations are useful because they describe the physics of many things of academic and economic interest. They may be used to model the weather, ocean currents, water flow in a pipe and air flow around a wing. The Navier–Stokes equations in their full and simplified forms help with the design of aircraft and cars, the study of blood flow, the design of power stations, the analysis of pollution, and many other things. Coupled with Maxwell's equations they can be used to model and study magnetohydrodynamics.

The Navier–Stokes equations are also of great interest in a purely mathematical sense. Somewhat surprisingly, given their wide range of practical uses, mathematicians have not yet proven that in three dimensions solutions always exist (existence), or that if they do exist, then they do not contain any singularity(smoothness). These are called the Navier–Stokes existence and smoothness problems. The Clay Mathematics Institute has called this one of the seven most important open problems in mathematics and has offered a US$1,000,000 prize for a solution or a counter-example.

Navier–Stokes equations *(general)*

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f},$$

where **v** is the flow velocity, ρ is the fluid density, *p* is the pressure, $\mathbf{T}$ is the (deviatoric) stress tensor, and **f** represents body forces (per unit volume) acting on the fluid and ∇ is the del operator. This is a statement of the conservation of momentum in a fluid and it is an application of Newton's second law to a continuum; in fact this equation is applicable to any non-relativistic continuum and is known as theCauchy momentum equation.

This equation is often written using the material derivative *D***v**/*Dt*, making it more apparent that this is a statement of Newton's second law:

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f}.$$

The left side of the equation describes acceleration, and may be composed of time dependent or convective effects (also the effects of non-inertial coordinates if present). The right side of the equation is in effect a summation of body forces (such as gravity) and divergence of stress (pressure and shear stress).

NAVIER–STOKES EXISTENCE AND SMOOTHNESS

The **Navier–Stokes existence and smoothness** problem concerns the mathematical properties of solutions to the Navier–Stokes equations, one of the pillars offluid mechanics (such as with turbulence). These equations describe the motion of a fluid (that is, a liquid or a gas) in space. Solutions to the Navier–Stokes equations are used in many practical applications. However, theoretical understanding of the solutions to these equations is incomplete. In particular, solutions of the Navier–Stokes equations often include turbulence, which remains one of the greatest unsolved problems in physics, despite its immense importance in science and engineering.

Even much more basic properties of the solutions to Navier–Stokes have never been proven. For the three-dimensional system of equations, and given some initial conditions, mathematicians have not yet proved that smooth solutions always exist, or that if they do exist they have bounded kinetic energy. This is called the*Navier–Stokes existence and smoothness* problem.

Since understanding the Navier–Stokes equations is considered to be the first step for understanding the elusive phenomenon of turbulence, the Clay Mathematics Institute offered a US$1,000,000 prize in May 2000, not to whoever constructs a theory of turbulence, but (more modestly) to the first person providing a hint on the phenomenon of turbulence. In that spirit of ideas, the Clay Institute set a concrete mathematical problem:

TWO SETTINGS: UNBOUNDED AND PERIODIC SPACE

There are two different settings for the one-million-dollar-prize Navier–Stokes existence and smoothness problem. The original problem is in the whole space $\mathbb{R}^3$, which needs extra conditions on the growth behavior of the initial condition and the solutions. In order to rule out the problems at infinity, the Navier–Stokes equations can be set in a periodic framework, which implies that they are no longer working on the whole space $\mathbb{R}^3$ but in the 3-dimensional torus $\mathbb{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$. Each case will be treated separately.

STATEMENT OF THE PROBLEM IN THE WHOLE SPACE

HYPOTHESES AND GROWTH CONDITIONS

The initial condition $\mathbf{v}_0(x)$ is assumed to be a smooth and divergence-free function

(see smooth function) such that, for every multi-index (see multi-index notation) and any $K > 0$, there exists a constant $C = C(\alpha, K) > 0$ (i.e. this "constant" depends on and *K*) such that

$$|\partial^\alpha \mathbf{v_0}(x)| \leq \frac{C}{(1+|x|)^K} \text{ for all } x \in \mathbb{R}^3.$$

The external force $\mathbf{f}(x,t)$ is assumed to be a smooth function as well, and satisfies a very analogous inequality (now the multi-index includes time derivatives as well):

$$|\partial^\alpha \mathbf{f}(x)| \leq \frac{C}{(1+|x|+t)^K} \text{ for all } (x,t) \in \mathbb{R}^3 \times [0,\infty).$$

For physically reasonable conditions, the type of solutions expected are smooth functions that do not grow large as $|x| \to \infty$. More precisely, the following assumptions are made:

1. $\mathbf{v}(x,t) \in \left[C^\infty(\mathbb{R}^3 \times [0,\infty))\right]^3, \qquad p(x,t) \in C^\infty(\mathbb{R}^3 \times [0,\infty))$
2. There exists a constant $E \in (0,\infty)$ such that $\int_{\mathbb{R}^3} |\mathbf{v}(x,t)|^2 dx < E$ for all $t \geq 0$.

Condition 1 implies that the functions are smooth and globally defined and condition 2 means that the kinetic energy of the solution is globally bounded.

THE MILLION-DOLLAR-PRIZE CONJECTURES IN THE WHOLE SPACE

(A) Existence and smoothness of the Navier–Stokes solutions in $\mathbb{R}^3$

Let $\mathbf{f}(x,t) \equiv 0$. For any initial condition $\mathbf{v}_0(x)$ satisfying the above hypotheses there exist smooth and globally defined solutions to the Navier–Stokes equations, i.e. there is a velocity vector $\mathbf{v}(x,t)$ and a pressure $p(x,t)$ satisfying conditions 1 and 2 above.

(B) Breakdown of the Navier–Stokes solutions in $\mathbb{R}^3$

There exists an initial condition $\mathbf{v}_0(x)$ and an external force $\mathbf{f}(x,t)$ such that there exists no solutions $\mathbf{v}(x,t)$ and $p(x,t)$ satisfying conditions 1 and 2 above.

HYPOTHESES

The functions sought now are periodic in the space variables of period 1. More precisely, let e_i be the unitary vector in the *i*- direction:

$$e_1 = (1,0,0)\,, \qquad e_2 = (0,1,0)\,, \qquad e_3 = (0,0,1)$$

Then $\mathbf{v}(x,t)$ is periodic in the space variables if for any $i = 1,2,3$, then:

$$\mathbf{v}(x + e_i, t) = \mathbf{v}(x,t) \text{ for all } (x,t) \in \mathbb{R}^3 \times [0,\infty).$$

Notice that this is considering the coordinates mod 1. This allows working not on the whole space $\mathbb{R}^3$ but on the quotient space $\mathbb{R}^3/\mathbb{Z}^3$, which turns out to be the 3-dimensional torus:

$$\mathbb{T}^3 = \{(\theta_1,\theta_2,\theta_3) : 0 \le \theta_i < 2\pi\,, \quad i = 1,2,3\}.$$

Now the hypotheses can be stated properly. The initial condition $\mathbf{v}_0(x)$ is assumed to be a smooth and divergence-free function and the external force $\mathbf{f}(x,t)$ is assumed to be a smooth function as well. The type of solutions that are physically relevant are those who satisfy these conditions:

Conditions:

3. $\mathbf{v}(x,t) \in \left[C^\infty(\mathbb{T}^3 \times [0,\infty))\right]^3\,, \qquad p(x,t) \in C^\infty(\mathbb{T}^3 \times [0,\infty))$

4. There exists a constant $E \in (0,\infty)$ such that $\int_{\mathbb{T}^3} |\mathbf{v}(x,t)|^2 dx < E$ for all $t \ge 0$.

THE PERIODIC MILLION-DOLLAR-PRIZE THEOREMS

(C) Existence and smoothness of the Navier–Stokes solutions in $\mathbb{T}^3$

Let $\mathbf{f}(x,t) \equiv 0$. For any initial condition $\mathbf{v}_0(x)$ satisfying the above hypotheses there exist smooth and globally defined solutions to the Navier–Stokes equations, i.e. there is a velocity vector $\mathbf{v}(x,t)$ and a pressure $p(x,t)$ satisfying conditions 3 and 4 above.

(D) Breakdown of the Navier–Stokes solutions in $\mathbb{T}^3$

There exists an initial condition $\mathbf{v}_0(x)$ and an external force $\mathbf{f}(x,t)$ such that there exists no solutions $\mathbf{v}(x,t)$ and $p(x,t)$ satisfying conditions 3 and 4 above.

MILLENNIUM PRIZE PROBLEMS

The **Millennium Prize Problems** are seven problems in mathematics that were stated by the Clay Mathematics Institute in 2000. As of September 2012, six of the problems remain unsolved. A correct solution to any of the problems results in a US$1,000,000 prize (sometimes called a *Millennium Prize*) being awarded by the institute. The Poincaré conjecture, the only Millennium Prize Problem to be solved so far, was solved by Grigori Perelman, but he declined the award in 2010.

The seven problems are:

1. P versus NP problem
2. Hodge conjecture
3. Poincaré conjecture (**solved**)
4. Riemann hypothesis
5. Yang–Mills existence and mass gap
6. Navier–Stokes existence and smoothness
7. Birch and Swinnerton-Dyer conjecture

QUOTIENT SPACE

Let X be a topological space, and let $\sim$ be an equivalence relation on X . Write X_* for the set ofequivalence classes of X under $\sim$. The QUOTIENT TOPOLOGY on X_* is the topology whose open sets are the subsets $U \subset X_*$ such that

$$\bigcup_{U \subset X}$$

is an open subset of X . The space X_* is called the QUOTIENT SPACE of the space X with respect to $\sim$. It is often written $X/\sim$.

The projection map $\pi: X \longrightarrow X_*$ which sends each element of X to its equivalence class is always acontinuous map. In fact, the map π satisfies the stronger property that a subset U of X_* is open if and only if the subset $\pi^{-1}(U)$ of X is open. In general, any surjective map $p: X \longrightarrow Y$ that satisfies this stronger property is called a QUOTIENT MAP, and given such a quotient map, the space Y is alwayshomeomorphic to the quotient space of X under the equivalence relation

$$x \sim x' \Longleftrightarrow p(x) = p(x').$$

As a set, the construction of a quotient space collapses each of the equivalence classes of $\sim$ to a single point. The topology on the quotient space is then chosen to be the strongest topology such that the projection map π is continuous.
For $A \subset X$, one often writes X/A for the quotient space obtained by identifying all the points of A with each other.

PARITY (PHYSICS)

Flavour quantum numbers:

- Isospin: **I** or I_3
- Charm: C
- Strangeness: S
- Topness: T
- Bottomness: B'

Related quantum numbers:

- Baryon number: B
- Lepton number: L
- Weak isospin: **T** or T_3
- Electric charge: Q
- X-charge: X

Combinations:

- Hypercharge: Y
  - $Y = (B + S + C + B' + T)$
  - $Y = 2\ (Q - I_3)$
- Weak hypercharge: Y_W
  - $Y_W = 2\ (Q - T_3)$
  - $X + 2Y_W = 5\ (B - L)$

Flavour mixing

- CKM matrix
- PMNS matrix
- Flavour complementarity

In quantum physics, a **parity transformation** (also called **parity inversion**) is the flip in the sign of*one* spatial coordinate. In three dimensions, it is also commonly described by the simultaneous flip in the sign of all three spatial coordinates:

$$P: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}.$$

It can also be thought of as a test for chirality of a physical phenomenon, in that performing a parity inversion transforms a chiral phenomenon into its mirror image. A parity transformation on something achiral, on the other hand, can be viewed as an identity transformation. All fundamental interactions of elementary particles are symmetric under parity, except for the weak interaction, which is sensitive to chirality and thus provides a handle for probing it, elusive as it is in the midst of stronger interactions. In interactions which are symmetric under parity, such as electromagnetism in atomic and molecular physics, parity serves as a powerful controlling principle underlying quantum transitions.

A 3×3 matrix representation of **P** would have determinant equal to −1, and hence cannot reduce to arotation which has a determinant equal to 1. The corresponding mathematical notion is that of apoint reflection.

In a two-dimensional plane, parity is *not* a simultaneous flip of all coordinates, which would be the same as a rotation by 180 degrees. It is important that the determinant of the P matrix be

−1, which does not happen for 180 degree rotation in 2-D, where a parity transformation flips the sign of *either x or y, but not both*.

SIMPLE SYMMETRY RELATIONS

Under rotations, classical geometrical objects can be classified into scalars, vectors, and tensors of higher rank. In classical physics, physical configurations need to transform under representations of every symmetry group.

Quantum theory predicts that states in a Hilbert space do not need to transform under representations of the group of rotations, but only under projective representations. The word *projective* refers to the fact that if one projects out the phase of each state, where we recall that the overall phase of a quantum state is not an observable, then a projective representation reduces to an ordinary representation. All representations are also projective representations, but the converse is not true, therefore the projective representation condition on quantum states is weaker than the representation condition on classical states.

The projective representations of any group are isomorphic to the ordinary representations of a central extension of the group. For example, projective representations of the 3-dimensional rotation group, which is the special orthogonal group SO(3), are ordinary representations of the special unitary group SU(2). Projective representations of the rotation group that are not representations are called spinors, and so quantum states may transform not only as tensors but also as spinors.

If one adds to this a classification by parity, these can be extended, for example, into notions of

- *scalars* ($P = 1$) and *pseudoscalars* ($P = -1$) which are rotationally invariant.
- *vectors* ($P = -1$) and *axial vectors* (also called *pseudovectors*) ($P = 1$) which both transform as vectors under rotation.

One can define **reflections** such as

$$V_x : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ y \\ z \end{pmatrix},$$

which also have negative determinant and form a valid parity transformation. Then, combining them with rotations (or successively performing *x*-, *y*-, and *z*-reflections) one can recover the particular parity transformation defined earlier. The first parity transformation given does not work in an even number of dimensions, though, because it results in a positive determinant. In odd number of dimensions only the latter example of a parity transformation (or any reflection of an odd number of coordinates) can be used.

Parity forms the Abelian group $\mathbb{Z}_2$ due to the relation $\mathbf{P}^2 = 1$. All Abelian groups have only one dimensional irreducible representations. For $\mathbb{Z}_2$, there are two irreducible representations: one is even under parity ($\mathbf{P}\varphi = \varphi$), the other is odd ($\mathbf{P}\varphi = -\varphi$). These are useful in quantum mechanics. However, as is elaborated below, in quantum mechanics states need not transform under actual representations of parity but only under projective representations and so in principle a parity transformation may rotate a state by any phase.

CLASSICAL MECHANICS

Newton's equation of motion $\mathbf{F} = m\,\mathbf{a}$ (if the mass is constant) equates two vectors, and hence is invariant under parity. The law of gravity also involves only vectors and is also, therefore, invariant under parity.

However, angular momentum **L** is an axial vector,

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$,

$\mathbf{P}(\mathbf{L}) = (-\mathbf{r}) \times (-\mathbf{p}) = \mathbf{L}$.

In classical electrodynamics, the charge density ρ is a scalar, the electric field, **E**, and current **j** are vectors, but the magnetic field, **H** is an axial vector. However, Maxwell's equations are invariant under parity because the curl of an axial vector is a vector.

EFFECT OF SPATIAL INVERSION ON SOME VARIABLES OF CLASSICAL PHYSICS

EVENCLASSICAL VARIABLES, PREDOMINANTLY SCALAR QUANTITIES, WHICH DO NOT CHANGE UPON SPATIAL INVERSION INCLUDE:

t, the time when an event occurs

m, the mass of a particle

E, the energy of the particle

P, power (rate of work done)

ρ, the electric charge density

V, the electric potential (voltage)

ρ, energy density of the electromagnetic field

$\mathbf{L}$, the angular momentum of a particle (both orbital and spin) (axial vector)

$\mathbf{B}$, the magnetic field (axial vector)

$\mathbf{H}$, the auxiliary magnetic field

$\mathbf{M}$, the magnetization

T_{ij} Maxwell stress tensor.

All masses, charges, coupling constants, and other physical constants, except those associated with the weak force

ODD

Classical variables, predominantly vector quantities, which have their sign flipped by spatial inversion include:

h, the helicity

Φ, the magnetic flux

$\mathbf{x}$, the position of a particle in three-space

$\mathbf{v}$, the velocity of a particle

$\mathbf{a}$, the acceleration of the particle

$\mathbf{p}$, the linear momentum of a particle

$\mathbf{F}$, the force exerted on a particle

$\mathbf{J}$, the electric current density

$\mathbf{E}$, the electric field

$\mathbf{D}$, the electric displacement field

$\mathbf{P}$, the electric polarization

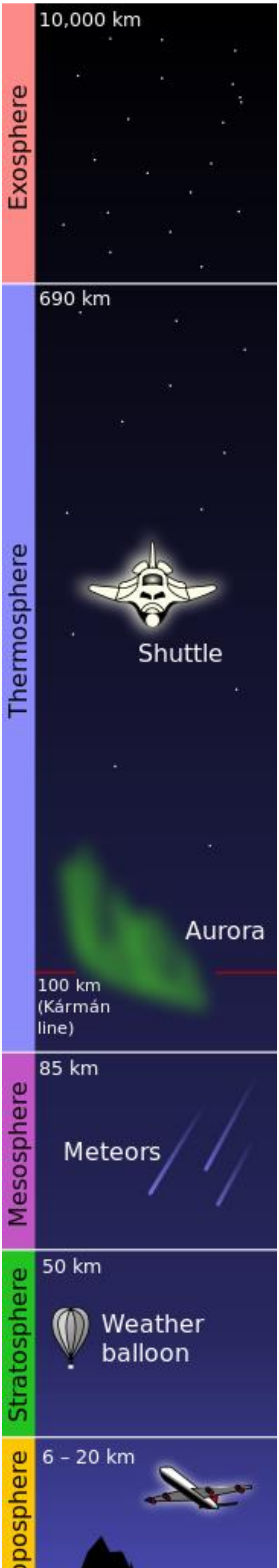
$\mathbf{A}$, the electromagnetic vector potential

$\mathbf{S}$, Poynting vector.

KÁRMÁN LINE

"Edge of space" redirects here. For the high-altitude region of Earth's atmosphere, see near space.

For the boundary of the universe, see observable universe.



Exosphere
10,000 km
Thermosphere
690 km
Shuttle
Aurora
100 km
(Kármán
line)
Mesosphere
85 km
Meteors
Stratosphere
50 km
Weather
balloon
6 – 20 km

Layers of Atmosphere.*(not to scale)*

The **Kármán line**, or **Karman line**, lies at an altitude of 100 kilometres (62 mi) above the Earth'ssea level, and commonly represents the boundary between the Earth's atmosphere and outer space. This definition is accepted by the Fédération Aéronautique Internationale (FAI), which is an international standard setting and record-keeping body for aeronautics and astronautics.

The line is named for Theodore von Kármán, (1881–1963) a Hungarian-American engineer andphysicist. He was active primarily in aeronautics and astronautics. He was the first to calculate that around this altitude, the atmosphere becomes too thin to support aeronautical flight, because a vehicle at this altitude would have to travel faster than orbital velocity to derive sufficientaerodynamic lift to support itself (neglecting centrifugal force. There is an abrupt increase inatmospheric temperature and interaction with solar radiation just below the line, which places the line within the greater thermosphere.

DEFINITION

An atmosphere does not abruptly end at any given height, but becomes progressively thinner with altitude. Also, depending on how the various layers that make up the space around the Earth are defined (and depending on whether these layers are considered part of the actual atmosphere), the definition of the edge of space could vary considerably: If one were to consider thethermosphere and exosphere part of the atmosphere and not of space, one might have to extend the boundary to space to at least 10,000 km (6,200 mi) above sea level. The Kármán line thus is an arbitrary definition based on the following considerations:

An aeroplane only stays in the sky if it is constantly traveling forward relative to the air (airspeed is not dependent on speed relative to ground), so that the wings can generate lift. The thinner the air, the faster the plane has to go to generate enough lift to stay up.

If the lift coefficient for a wing at a specified angle of attack is known (or estimated using a method such as thin-airfoil theory), then the lift produced for specific flow conditions can be determined using the following equation

$$L = \frac{1}{2}\rho v^2 A C_L$$

where

L is lift force

ρ is air density

v is speed relative to the air

A is wing area,

C_L is the lift coefficient at the desired angle of attack, Mach number, and Reynolds number.

Lift (L) generated is directly proportional to the air density (ρ). All other factors remaining unchanged, true airspeed (v) must increase to compensate for less air density (ρ) at higher altitudes.

An orbiting spacecraft only stays in the sky if the centrifugal component of its movement around the Earth is enough to balance the downward pull of gravity. If it goes slower, the pull of gravity gradually makes its altitude decrease. The required speed is called *orbital velocity*, and it varies with the height of the orbit. For the International Space Station, or a space shuttle in low Earth orbit, the orbital velocity is about 27,000 km per hour (17,000 miles per hour).

For an aeroplane flying higher and higher, the increasingly thin air provides less and less lift, requiring increasingly higher speed to create enough lift to hold the aeroplane up. It eventually reaches an altitude where it must fly so fast to generate lift that it reaches orbital velocity. The Kármán line is the altitude where the speed necessary to aerodynamically support the aeroplane's full weight equals orbital velocity (assuming wing loading of a typical aeroplane). In practice, supporting full weight wouldn't be necessary to maintain altitude because the curvature of the Earth adds centrifugal lift as the aeroplane reaches orbital speed. However, the Karman line definition ignores this effect because orbital velocity is implicitly sufficient to maintain any altitude regardless of atmospheric density. The Karman line is therefore the highest altitude at which orbital speed provides sufficient aerodynamic lift to fly in a straight line that doesn't follow the curvature of the Earth's surface.

When studying aeronautics and astronautics in the 1950s, Kármán calculated that above an altitude of roughly 100 km (62 mi), a vehicle would have to fly faster than orbital velocity to derive sufficient aerodynamic lift from the atmosphere to support itself. At this altitude, the air density is about 1/2200000 the density on the surface. At the Karman line, the air density ρ is such that

$$L = \frac{1}{2}\rho v_0^2 A C_L = mg$$

where

v_0 is orbital velocity

m is mass of the aircraft

g is acceleration due to gravity.

Although the calculated altitude was not exactly 100 km, Kármán proposed that 100 km be the designated boundary to space, since the round number is more memorable, and the calculated altitude varies minutely as certain parameters are varied. An international committee recommended the 100 km line to the FAI, and upon adoption, it became widely accepted as the boundary to space for many purposes.[5] However, there is still no international legal definition of the demarcation between a country's air space and outer space.

Another hurdle to strictly defining the boundary to space is the dynamic nature of Earth's atmosphere. For example, at an altitude of 1,000 km (620 mi), the atmosphere's density can vary by a factor of five, depending on the time of day, time of year, AP magnetic index, and recent solar flux.

The FAI uses the Kármán line to define the boundary between aeronautics and astronautics:

Aeronautics — For FAI purposes, aerial activity, including all air sports, within 100 kilometres of Earth's surface.

Astronautics — For FAI purposes, activity more than 100 kilometres above Earth's surface.

INTERPRETATIONS OF THE DEFINITION

Some people (including the FA(in some of their publications) also use the expression "**edge of space**" to refer to a region below the conventional

100 km boundary to space, which is often meant to include substantially lower regions as well. Thus, certain balloon or airplane flights might be described as "reaching the edge of space". In such statements, "reaching the edge of space" merely refers to going higher than average aeronautical vehicles commonly would.

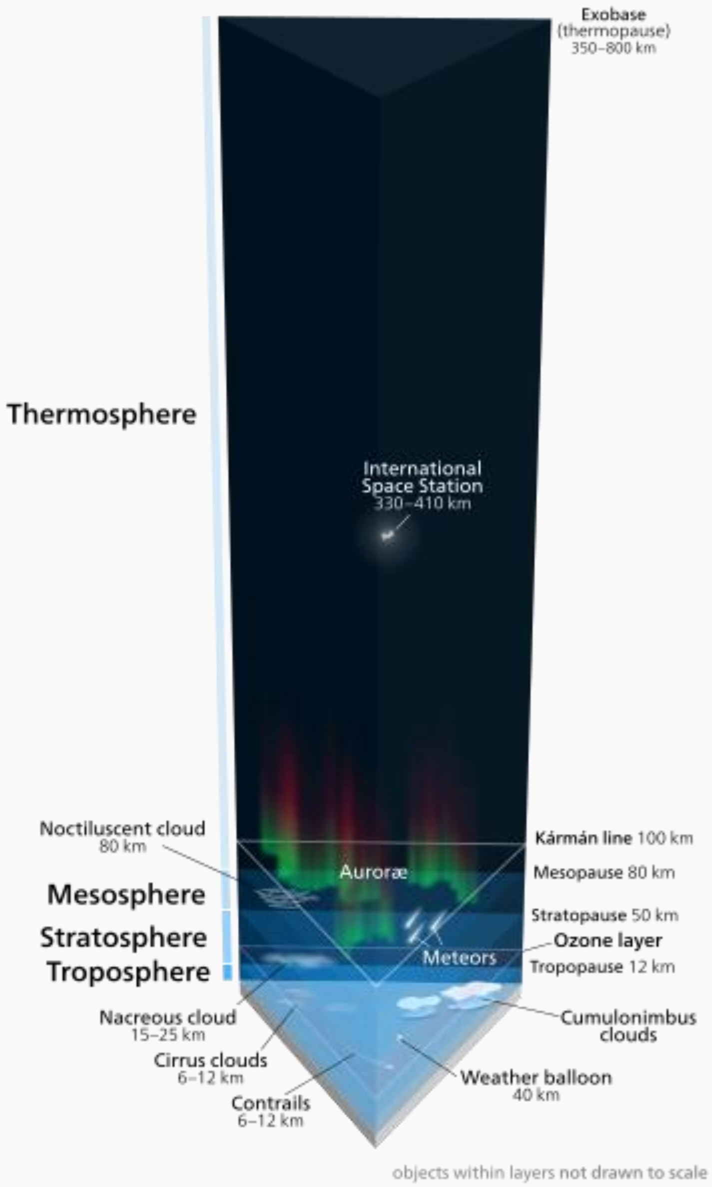
ALTERNATIVES TO THE DEFINITION

Although the United States does not officially define a *boundary of space*, the U.S. definition of an astronaut, which is still held today, is a person who has flown more than 50 miles (~80 km) above mean sea level. (This is approximately the line between themesosphere and the thermosphere.) This definition of an astronaut had been somewhat controversial, due to differing definitions between the United States military and NASA.

In 2005, three veteran NASA X-15 pilots (John B. McKay, Bill Dana and Joseph Albert Walker) were retroactively (two posthumously) awarded their astronaut wings, as they had flown between 90 and 108 km in the 1960s, but at the time had not been recognized as astronauts.

International law defines the lower boundary of space as the lowest perigee attainable by an orbiting space vehicle, but does not specify an altitude. Due to atmospheric drag, the lowest altitude at which an object in a circular orbit can complete at least one full revolution without propulsion is approximately 150 km (93 mi), while an object can maintain an elliptical orbit with perigee as low as 129 km (80 mi) with propulsion.

Atmospheric gases scatter blue wavelengths of visible light more than other wavelengths, giving the Earth’s visible edge a blue halo. At higher and higher altitudes, the atmosphere becomes so thin that it essentially ceases to exist. Gradually, the atmospheric halo fades into the blackness of space.



$$\mathbf{P}\,\Psi = c\,\Psi,$$

$$\mathbf{P}^2\Psi = c\mathbf{P}\ \Psi.$$

The intrinsic parity assignments in this section are true for relativistic quantum mechanics as well as quantum field theory.

$$\mathbf{P}\mathbf{a}(\mathbf{p}, \pm)\mathbf{P}^{+} = -\mathbf{a}(-\mathbf{p}, \pm)$$

$$\mathbf{P}\mathbf{a}(\mathbf{p})\mathbf{P}^{+} = \mathbf{a}(-\mathbf{p}).$$